THEORY OF REACTION-DIFFUSION AND EMERGENCE OF THE GEOGRAPHICAL FORMS

André Dauphiné
Université de Nice Sophia-Antipolis

Geography studies the organisation of territories in their physical dimensions as well as in their social dimensions. This is thus a knowledge about emergence of forms such as the scattering of fallows and then forests over abandoned agricultural soils, the growth of a city or of a transport network, or the spreading of a pioneer front in Brazil or Siberia.

In order to account for the emergence of spreading or stationary forms, a lot of scientists from other disciplines often refer to a theory simultaneously suggested by Fisher, Kolmogorov, Petrovskii and Piskunov, and usually designated by the acronym FKPP. Numerous papers and books dedicated to this theory and to the models derived from, such as Turing activator-inhibitor model or their generalisation by Meinhardt. As the creation of a chemical, its production and growth in time, is usually obtained by reaction, this theory is generally called « theory of reaction-diffusion ». We have highlighted the interest of this theory since 1985, at the European Colloquium on Theoretical and Quantitative Geography. But it was not a source of inspiration for the geographers.

We suggest then to insist on the components that should be added to make this theory effective in geography. Beyond the long distance interactions designated as convection, advection, turbulence in nature sciences or transport in the societal sphere, it is advisable to think about the initial conditions that strongly influence the emergence of the geographical forms and more about the introduction of human actors adaptability.

1. A theory very quickly enriched

1.1. A centenary theory

The theory of reaction-diffusion resumes Heraclite's idea for who every creation of a form is the product of the struggle between two principles. In its simplest form, this theory is illustrated by an equation which is written as following in usual language:

Variation of a = Growth of a - Diffusion of a

In a given point of space, the law of reaction, production or growth integrates the creation and the disappearance of new « individuals », molecules in chemistry, cells in biology, individuals in ecology, or persons in social sciences. Frequently, this growth process is auto-catalytic: the fabrication of a product « x » depends on the presence of « x » and usually on its density. The autocatalyse is actually synonym of positive retroaction. For example, the demographic growth is function of the number of reproductive couples and, then, of the population size.

In this theory, the role of diffusion becomes anti-intuitive. As a rule, diffusion restores equilibria in a gradient field. It homogenizes this field and tends to erase forms. Coupled to a growth mechanism, diffusion has therefore a reverse effect and generates new forms. These forms are created and remained because diffusion is constantly feeded by the growth mechanism.

Following the considered discipline, this equation or system of equations, when at least one of both components is considered, has different names. In chemistry, it stands for a process of reaction-diffusion illustrated by the experiment of Belousov-Zabothinski (BZ). In ecology, this theory is named by different terms because ecologists use different growth models. They mention a KISS model (Kierstead, Slobodkin et Skellam) when growth is represented by an exponential law. The corresponding model is often solicited to explain the emergence and size of marine phytoplankton. In biology, this theory originates the activator-inhibitor model first figured out by Alan Turing in 1952. Actually, this theoretical model is well adapted to every statistical population composed by individuals that reproduce and diffuse in space.

1.2. Three types of generalisation

This theory, illustrated by an equation which integrates both mechanisms of growth and diffusion, can be generally applied. To make it applicable to the study of more realistic issues, three ways have been explored: modifying terms of growth or diffusion, taking into account several populations in interaction and finally, studying a mechanism additional to the initial equation.

A first form of generalisation consists in modifying the growth or diffusion mechanism. Indeed, there is not one but several growth laws. Thus, the observed growths in human societies are rarely exponential. The logistical Verhulst law, more realistic than the exponential growth, simulates a growth slowed down by density. This slowing down applies to most of the living populations which have a restricted food available. In ecology, the model of reaction-diffusion, built from this logistical growth, was qualified as the Fisher model because of the name of the mathematician who introduced it since 1934. This model will be then applied to represent the agriculture emergence in Europe in the Neolithic.

This first generalisation sometimes applies on the second mechanism, the diffusion. In its simple form, diffusion depends on the difference of the concentration of the considered product in space. Diffusion transfers the product from high densities to low densities. It is a slow process because it acts only gradually by contiguity. And the diffusion coefficient is often assimilated to a constant. But, it is possible to make the diffusion coefficient vary, for example in function of the density. Indeed, high densities fasten the diffusion process as testify the studies carried out on migratory fluxes: migrants rather leave very peopled spaces.

A second type of generalisation is obtained by adding new terms to the elementary equation in order to take other factors into account. Thus, a city grows up and spreads by the growth of its population and its diffusion in suburbs. But, in the same time, the centre, as well as the peripheral spaces, receives a demographic excess from the rural environment, other cities or further countries. A migratory phenomenon is superimposed on auto-growth and diffusion and is easily assimilated to an advection. To insert this new mechanism, it is sufficient to insert this advection mechanism in the initial equation, which then can be written as following:

Variation of a = Growth of a - Diffusion of a + Advection of a

This advection is symbolised, as diffusion, by a partial derivative affected of an advection coefficient which obviously does not have the same value as the diffusion coefficient. Thus, both elementary mechanisms are conserved but they are amplified or thwarted by these larger moves.

A third generalisation, already announced for the BZ model, consists on coupling two or several equations. Indeed, in the reality, a species never lives alone in its environment or its ecological niche. In physics and chemistry, molecules and products mix together. And, in geography or in social sciences, these interactions between different human groups are often more frequent. In a city, several ethnic groups or social classes cooperate or confront each other. In numerous disciplines, these interactions concern two populations. The ecologist analyses relations between preys and their predators from Volterra studies. Biologists who are specialists of animal forms development use the same equations. But they talk about activator-inhibitor model (Hans Meinardth, 1982). For two populations, the Turing theory is then expressed by a model which includes both of the following equations:

$$\begin{cases} \textit{Variation of a = Growth of a - Diffusion of a + Interaction of a and b} \\ \textit{Variation of b = Growth of b - Diffusion of b + Interaction of b and a} \end{cases}$$

Each equation includes both mechanisms of the basis model, simulating growth and diffusion and also an interaction mechanism. In this system of two equations, it is easy, in function of the positive or negative value attributed to interactions, to represent an effect of competition or, inversely, an effect of mutualism, of cooperation between the activator and the inhibitor, between both species. Fighting is not always the rule even if it was considered as essential by Malthus, Darwin and Marx.

This form of generalisation can be applied beyond two populations. Epidemiologists also use these equations by distinguishing sane and infected populations and populations which re-cover or disappear. They build then SIR (Susceptible, Infected, Release) models which simulate the behaviour of three populations. Recently, Michaël Batty (1999) adopted this model to represent dynamics of urban growths.

This theory of reaction-diffusion generates an infinity of forms. The combination of the simple laws is sufficient to simulate the emergence of linear forms, networks, gradient forms, repetitive forms, spiral forms of cellular textures, hexagonal for example. This set of two laws succeeds in creating homogeneous or heterogeneous forms, isotropic or anisotropic, punctual or areolar. Actually, as highlighted by James Murray (1990): « The theory of reaction-diffusion predicts a very rich diversity of complex forms, from the simplest as invasion fronts and solitary waves, to the most complex as spirals and spatio-temporal chaos ». The first works were about fronts dynamics. More recent studies insist on the persistence and stable condition of some forms. Thus, strips and hexagons are more stable than other forms. This attests of their higher frequency in Nature. Biologists are the ones who best studied the laws which govern forms generated by these mechanisms. Usually, forms stabilise when the diffusion coefficient of the inhibitor is seven times higher than the activator's one. In function of the size of the considered space and conditions of production, diffusion and interaction, several similar forms can appear at more or less regular space intervals.

2. Two geographical constraints: initial conditions and role of the actors

To apply the theory of reaction-diffusion in geography, a double enrichment is imperative. It concerns the initial conditions and the role of the social actors who have an amazing capacity of adaptation.

2.1. Initial conditions often determining

The initial conditions considered by physicians are usually simple. These mechanisms act on homogeneous spaces, isotropic or random. But, in geography, the terrestrial space is always structured and heterogeneous. Agents do not interact from random or homogeneous distributions. In a river, a pollutant will spread in function of the pressure gradient (diffusion) and the associated wind (advection) but also in function of the underlying topography. In a city, people diffusion takes road networks. All preliminary structures of the terrestrial space, physic or human, channel diffusion and advection mechanisms and they also intervene on growth.

It is then imperative to take into account this ground reality by introducing this terrestrial pre-structure in the initial conditions and then by qualifying the associated effects. Works of that type, developed by physicians, can guide geographers such as, for example, the book of Ben-Avraham and Havlin (2000) dedicated to the study of the mechanisms of reaction and diffusion in heterogeneous and fractal environments. They show how pre-existing structures have a direct influence on the emergence of new forms. This is a source of inspiration for geographers.

2.2. The agents of human societies are adaptive

A second specificity of geographic forms lies in the originality of the human action. Humans are not simple molecules. They pursue goals and their actions tend to reach economical, social or cultural objectives. It is then necessary to include these rules of behaviours in the theory of reaction-diffusion, rules which will depend on the scientific issue considered. These rules of behaviours are very numerous but can be reduced to some generic laws.

Humans act by comparing themselves to others but also by learning, by taking the past into account and anticipating the future. Thus, in a research about the emergence of segregation forms inside a city, Thomas Schelling showed that diffusive moves of each person are made after comparison of his cultural values with his neighbours' ones. Beyond a threshold of dissimilarity which can be modified in simulations, people leave their home and move to people sharing the same cultural values. In the course of time, people who have similar values gather and homogeneous quarters emerge from a random initial distribution. Then, the setting up of a city with its segregation phenomena needs no chief, no villain as suggested by some theories.

But, this initial model deserves to be deepened. Originally, both populations have a fixed and constant size. It is thus necessary to introduce the real growth of these populations to obtain a simple model of reaction-diffusion. Then, it is desirable to include, not one but several types of ethnic or social classes. Geographers make then a double segregation emerge which show homogeneous quarters but also gatherings always similar to these homogeneous quarters.

Segregation thus appears at two different spatial scales : green with green but also quarters which became green still stay in contact with blue quarters. This pattern corresponds to the urban organisation of great American cities.

Besides, in contact with a population which shows different value, a person can evolve, learn and adopt these values. This learning is frequent when persons in contact have different social status. It is assumed that classes said to be « underprivileged » tend to adopt quite easily values or behaviours of classes qualified as « well-off ». More globally, numerous socio-cultural criteria evolve under the neighbourhood pressure whereas others, such as religious believes, are very stable.

Contrarily to gaseous molecules, humans can even determine neighbours' choice with who they compare each other and interact and they can even take into account the past interactions whether negative or positive. This is namely the case of the models built on the prisoner's dilemma. Rules of decision and interaction are then implemented in the models of reaction-diffusion. Numerous simulation models illustrating the results of these behaviours are already available (Richard Gaylord and Louis d'Andria, 1998). They generally include a moving process by diffusion under the form of a brownian move.

Eventually, numerous human moves are made on long distances and are not assimilable to diffusion any more. Couples leave their city, their region or their country. An advection mechanism should be added to the diffusive moves and the brownian move should be replaced by a Levy flight.

Obviously, if this generalised cultural process of reaction-diffusion generates socio-spatial segregation, the accurate location of the quarters is often arbitrary and changes in function of the initial conditions in the simulation models. This is not the case in real cities where land-prices often determine these locations. In French Mediterranean cities, Italian quarters of the Fifties and actual North African quarters are rather located in wet glens devoid of a large view over the sea.

CONCLUSION

In order to understand the emergence of forms in geography, the theory of « Reaction-Diffusion » is an excellent starting point. However, the geographer must choose the most relevant growth (reaction) model. Moreover, he must add an advective component because only diffusive moves of human order are exceptional. Humans as albatross have daily moves which obey more generally to the formalism of a Levy flight, integrating short moves and further moves. Besides, the weight of the initial conditions is essential. Diffusion said to be normal is exceptional. Whatever the fluxes of matter, energy, people, money or information which generate territorial forms, they diffuse over a non-homogeneous space which imposes its structure. Real forms are not « free » but their emergence is channelled by these initial conditions and at the limits. In addition, human actions and interactions obey to a principle of adaptation including memory, learning and then evolutionary behaviours. This diversity does really not condemn an approach by the model of reaction-diffusion. It adds a source of complexity in the research in social sciences.

References

Ben-Avraham D., Havlin Sh., 2000, Diffusion and Reactions in Fractals and Disordered Systems, Cambridge, Cambridge University Press, 316 p.

Batty M., 2005, Cities and Complexity, Cambridge, MIT Press, 565 p.

Bourgine P., Lesne A., 2006, Morphogenèse, L'origine des formes, Paris, Belin, 352 p.

Cantrell R., Cosner Ch., 2003, Spatial Ecology via Reaction-Diffusion Equations, Chichester, John Wiley & Sons, 411 p.

Dauphiné A., 2003, Les théories de la complexité chez les géographes, Paris, Economica, 248 p.

Gaylord R., d'Andria L., 1998, Simulating Society, New York, TELOS, 218 p.

Grindrod P., 1996, The Theory and Applications of Reaction-Diffusion Equations, Oxford, Clarendon Press, 275 p.

Livi R., Vulpiani A., 2003, L'héritage de Kolmogorov en physique, Paris, Belin, 254 p.

Meinhardt H., 1995, The Algorithmic Beauty of Sea Shells, Berlin, Springer, 204 p.

Murray J., 2003, Mathematical Biology tome II Spatial Models and Biomedical Applications, New York, Springer, 811 p.

Okubo A., Levin S., 2001, Diffusion and Ecological Problems, New York, Springer, 467 p.

Rabino G., 2005, Processi Decisionali e Territorio nella Simulazione Multi-Agente, Bologna, Esculapio, 210 p.

Zwirn H., 2006, Le s Systèmes Compexes, Paris, odile Jacob, 219 p.