Mean Degree of Ad Hoc networks in Environments with Obstacles

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***Abstract*—What could be de mean degree of an ad network within a limited size environment containing obstacles? Does it depend on the number of obstacles? On their size? On their surface? On their position? All together? In this work we answer these questions and to others. We provide a method for computing the mean degree of an ad hoc network in a limited size rectangular area with the presence of obstacles. We show that, under a restricted set of assumptions, the value of the mean degree may be computed using a generic formula which parameters depend on the size of the environment and on various geometric characteristics of the obstacles.**

I. Introduction

Let us consider a set of wireless-enable communicating devices, later called stations, nodes or sensors according to the context. If these stations are able to self-organized themselves in order to build a communication network without relying on any infrastructure, they form an ad hoc network. Neither supervision nor centralized control is necessary for building the network. In this work we propose to study topological properties of the underlying communication graph. Such kind of studies may help understanding the performances of algorithms operating on them.Such studies may produce key results for calibrating wireless sensor networks in order to obtain, with high probability, connected networks after deployment or may provide some information from which the lifetime of these sensor networks can be estimated.

In this paper we propose a method for computing the mean degree of such networks, a first step in the direction of estimating their probability to be connected. The considered environment of deployment is a square or a rectangular area containing obstacles. These environments represent simplified models of urban areas that may be considered as the most natural environments for such networks. Indeed, nowadays, because of both economical and environmental preoccupations, many sensors are present in city centers. Moreover, a large portion of the population and a growing number of cars are equipped with smartphones or others wireless-enabled communicating devices, and all these materials could technically self-organize to form an ad hoc network.

This work has been motivated by the emergence of two tendancies. On the one hand, people studying ad hoc and mobile ad hoc networks have tried to take into account obstacles in their models of ad hoc networks. On the other hand, many works have been devoted to the study of connectivity properties of ad hoc networks. But, so far, no work has been proposed for computing connectivity evidences in the case of ad hoc networks embedded in environments with obstacles. This work is a first attempt in that direction.

Most of existing works dealing with ad hoc networks with obstacles focus on the mobility of stations, moreover, two types of obstacles are generally considered. The first one corresponds to non solid obstacles; areas where no station can be positionned but that do not prevent the propagation of the signal. Thus, only the movements of stations are constrained by these areas. Manhattan, freeway or pathway mobility models [2] fall in that category. The second type of obstacles are solid ones. In that case, not only the movement of stations but also the propagation is constrained by the obstacles. For the movement of stations, many scenarios have been developed to avoid them. For instance, in [15] [12] nodes move according to a Voronoi tessellation. For the coverage, obstacles can impact directly nodes coverage and influence their connectivity. This impact depends on the permeability of wall of obstacles. Recent works address this problem. In [**?**], the propagation is in line-of-sight *i.e* there is a link between two stations if and only if they are mutually and linearly visible. The environment is a torus composed of Manhattan lines. The coverage of nodes is a distance and the connectivity is approximated by considering an infinite number of nodes.

II. Models et Method

A. Models

We consider three models: a model of signal propagation, a model of station and a model of environment.

All stations are supposed homogeneous and equipped with omnidirectional antenna. In addition they all have the same power for emitting signal. Such stations are randomly and uniformly distributed in the environment. After beeing dropped or positionned within the environment each station cannot move. The signal propagation model considered is the pathloss model, the signal is attenuated with the distance, and it also follows the restriction of line-of-sight propagation, meaning that the signal is unable to pass through obstacles. Thus, each station is then characterized by a transmission range r and its coverage by a disk, centered at the station position with a radius equals to r. Communication between stations depend on their euclidian distance. If the distance between two stations is greater than a given value (the threshold), then no communication can occur between them. On the contrary, if the distance is lower than the threshold (r) and if the stations are in line-of-sight of each other, then the communication can occur.

For the environment, we consider a limited, and bounded (not wraparound), rectangular area of size $L \times l$. Within this environment some obstacles are present. Obstacles may be simple rectangles or may have more complex shape corresponding to overlapping rectangles whose sides are parallel to the borders, and whose edges are at least at long as twice the transmission range of the stations, denoted as $\Delta = 2r$ (see Figure 1). Stations cannot be located inside the obstacles, and the signal cannot pass through them. Finally, the distance between two obstacles, or between the border and an obstacle, has to be greater than or equal to Δ (twice the transmission range of the stations).

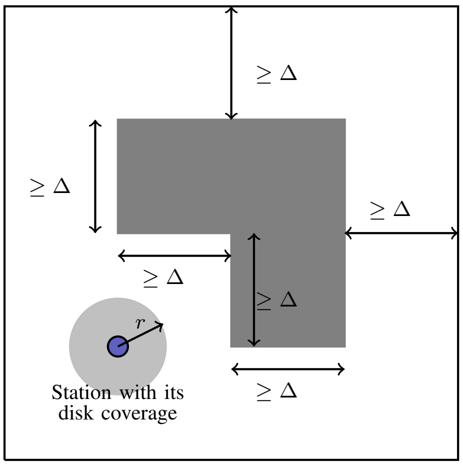


Fig. 1: Obstacles may have different shapes obtained from overlapping rectangles. However, some size and distance constraints have to be respected. Each edge of an obstacle has to be greater than $\Delta = 2r$ (twice the transmission range). The distance between two obstacles or between an obstacle and the border of the environment has also to be greater than Δ.

For computing the mean degree of the communication graph of the ad hoc network, we consider the underlying graph. Each node corresponds to a station and two nodes are linked by an edge if and only if the euclidian distance between the stations is lower than r and if they are in line-of-sight of each other. As the transmission range of all the stations is the same, and if we ignore obstacles, our graphs are similar to unit disk graphs [8]. More generally, this type of graphs used for representing ad hoc networks belongs to the category of random geometric graphs as they were defined by Penrose [19]. Such graphs are made of vertices that are randomly, uniformly and independently, placed on a k-dimensional space, and two vertices are linked by an edge if and only if the distance between them is lower than or equal to a given threshold r. In our context, we only consider a limited 2-dimensional space and obstacles may prevent two vertices to be linked even if the distance between them is lowen than or equal to a given threshold r.

B. Method

In most works on ad hoc networks, the method for determining connectivity properties relies on the fact that nodes are distributed according to a Poisson point distribution obtained as the result of the convergence of a binomial law when the number of stations/vertices is high enough and when the probability of finding a station within the transmission range of another one has a low probability value. More formally, if we consider a set of n vertices in an area without border but limited in space, then the probability for a node to have a given number of neighbors equals to k is equal to the probability of having k nodes among $n-1$ included in its communicating or coverage disk, whatever the stations. As the position of each station is supposed independent, and if we denote by p the probability of beeing in the disk, the probability for a vertex to have a degree equals to k is equal to:

$$P(d(v) = k) = \left(\begin{array}{c} n-1 \\ k \end{array} \right) p^k (1-p)^{n-1-k} \qquad (1)$$

For n large enough and for small values of the ratio coverage disk over total surface of the considered area, we can give an approximate solution since the binomial law converges toward a Poisson law if the previous conditions are verified. Its general form is:

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!} \qquad (2)$$

where λ is the parameter of the Poisson law and is also the *mean degree* of the vertices of the network.

For environments with obstacles, expressing the probability of a vertex to have a given number of neighbors in order to obtain an expression that could be identified as a known probability law is unlikely. Indeed, Bettstetter in [3] noticed that there is a significant quantitative difference between analytical values and simulated ones due to the border effect. There exist at least two main ways for avoiding this problem. One consists in considering a toroïdal distance instead of the euclidean one, which prevents the borders to impact the coverage of the stations. The second possibility consists in dividing the simulated area in two parts, an inner part and a border part, and only the nodes located within the inner part are considered for the analyses. But none of these solutions can be applied in our context since the borders of the obstacles cannot be avoided or masked within the simulation.

However, approximating the mean degree of a random geometric graph in environments with obstacles remains possible. The method consists in identifying in an environment with obstacles what are the different zones for which the coverage has to be formulated differently. For each zone some geometric computations are performed in order to compute the mean value of the coverage. Finally we combine the results in order to obtain the value of the mean degree of the graph. We first begin with an obstacle-free environment for measuring only the impact of the border on the mean degree (Section III). In a second step, we add one rectangular obstacle and we show that the impact of the obstacle differs according to its position within the environment (Section IV). Finally we propose a generalization for environments containing many obstacles with possibly various shapes (Section V).

III. Environments without Obstacles

As it was underlined by many authors in the litterature, the borders have an impact on the connectivity properties of ad hoc networks deployed within bounded areas. In our method, the first step for computing the mean number of neighbors of stations in such networks consists in identifying the different zones for which the coverage of stations varies. We consider a rectangular area of surface S in which a set of n stations are randomly and uniformly distributed. Let us denote (x, y) the set of coordinates in the rectangular environment, where the $(0, 0)$ point corresponds to a corner, and $p(x, y)$ the probability of presence of a station at this position. Let δ be the station density in the environment, $\delta = \frac{n}{S}$ and c be the coverage of a station. Because of the border, c depends on the euclidean distance of the station to the border. If this distance is greater than or equal to r then $c = \pi r^2$, but if this distance is lower than r, then $c = c(x, y)$ where (x, y) denotes the position of the station in the rectangular environment. The degree d of a station is the number of neighbors located in his coverage area.

$$d(x, y) = \delta \times c(x, y)$$

In an obstacle-free environment, we can distinguish four different zones, identified as zone 1, 2, 3 and 4 as illustrated by Figure 2. In zone 3, the coverage corresponds to a disk of radius r and in the three other ones to a non complete disk. In zone 1, 2 and 4, the coverage of a station depends also on its position within the zone, however it is possible to compute the mean value of the coverage of the stations located in a given zone.

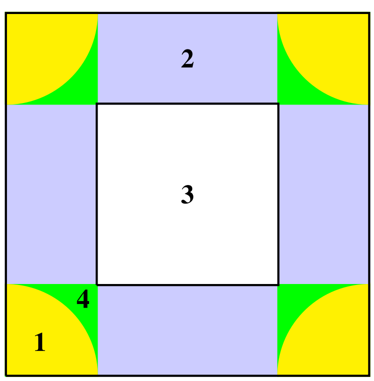


Fig. 2: The four zones of the environment.

The node coverage c depends on the node position in the environment. Let i be a given zone in the environment ($i \in \{1, 2, 3, 4\}$). C_i is the sum of coverages of nodes in the zone i:

$$C_i = \int_{x_0}^{x_1} \int_{y_0}^{y_1} c_i(x, y) dx dy$$

The mean coverage $\overline{C_i}$ for a given zone i is:

$$\overline{C_i} = \sum_{(x,y) \in i} p(x, y) \times c(x, y)$$

Let S_i be the sum of zones i in the environment. As the distribution of stations in the environment is uniform, the probability for a node to be in zone i is then $p_i = \frac{S_i}{S}$. The mean degree D of nodes in an environment containing k zones is:

$$D = \delta \times \sum_{i=1}^{k} \left[\frac{S_i}{S} \times \overline{C_i} \right] \tag{3}$$

Then, the mean degree depends on the density of nodes in the environment, on the surface of the environment in which the nodes are positioned and on the mean coverage of the nodes in every zone.

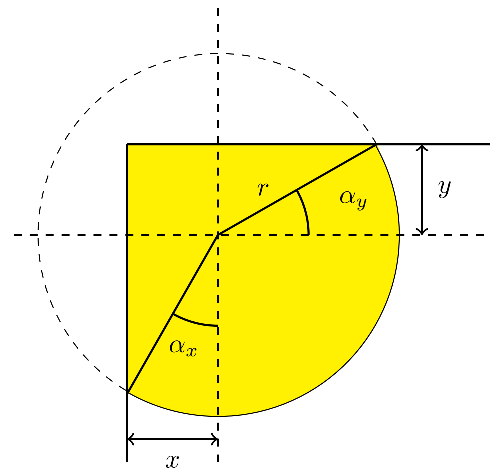


Fig. 3: Coverage of a station located in zone 1.

The *zone* 1 correspond to the quarter of the disk located in the corner of the environment (see Figure 3). The coverage of a node positioned in the *zone* 1 is given below:

If $\alpha_x = \arccos(\frac{x}{r})$ and $\alpha_y = \arccos(\frac{y}{r})$ (see Figure 3):

$$\begin{aligned} c_1(x, y) &= x.y + 3.\pi.r^2/4 - r^2.\alpha_x/2 - r^2.\alpha_y/2 \\ &\quad + x.r.\sin(\alpha_x)/2 + y.r.\sin(\alpha_y)/2 \end{aligned}$$

then

$$C_1 = \int_0^r \int_0^{\sqrt{r^2 - y^2}} c_1(x, y) dx dy$$

and: $\overline{C}_1 = (\pi^2 + 1) r^2 / (2\pi)$.

The same kind of computations can be performed for the other zones and then:

$$\begin{aligned} \overline{C}_1 &= (\pi^2 + 1) r^2 / (2\pi) \\ \overline{C}_2 &= (3\pi - 2) r^2 / 3 \\ \overline{C}_3 &= \pi.r^2 \\ \overline{C}_4 &= r^2 (24\pi - 32 - 3\pi^2) / (6(4 - \pi)) \end{aligned}$$

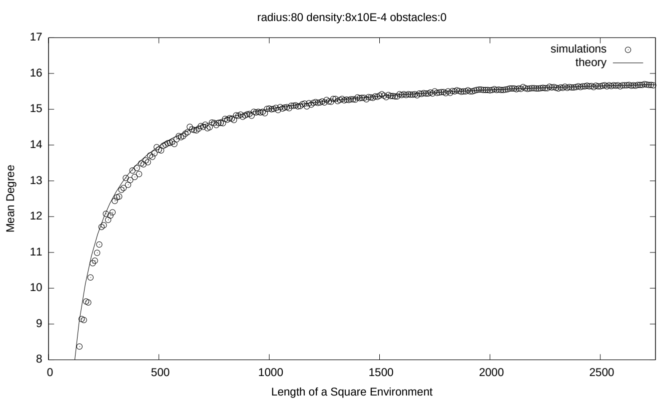
Let s_i $(1 \leq i \leq 4)$ be the surface of one occurrence of a zone i. We have:

$$\begin{aligned} s_1 &= \pi r^2/4 \\ s_2 &= r(L-2r) \text{ or } r(l-2r) \\ s_3 &= (L-2r)(l-2r) \\ s_4 &= r^2(4-\pi)/4 \end{aligned}$$

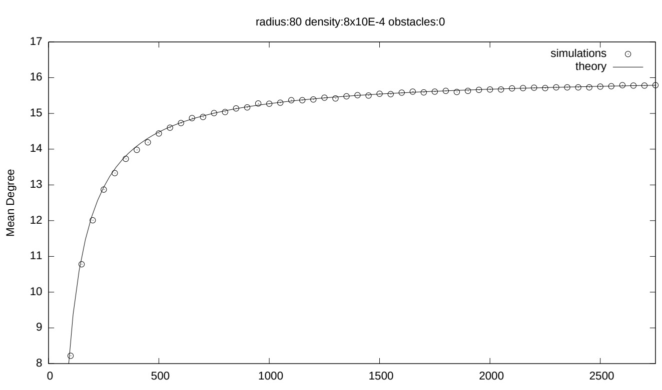
It comes that the cumulative surface of each type of zone i, denoted by S_i $(1 \leq i \leq 4)$is:

$$\begin{aligned} S_1 &= 4 \times s_1 = \pi r^2 \\ S_2 &= 2 \times (r(L-2r) + r(l-2r)) = 2r(L+l-4r) \\ S_3 &= (L-2r)(l-2r) \\ S_4 &= 4 \times s_4 = r^2(4-\pi) \end{aligned}$$

The mean degree of an environment without obstacle is then given by formula 3 for which all the terms have been determined in this section.



(a) Square environment



(b) Rectangular environment

Fig. 4: Evolution of mean degree in a square and a rectangular environments. The minimum size of the square environment is $2r \times 2r$. For lower sizes, areas 2 are overlapping, and the formula are no longer valid. For the rectangular environment, the length is the double of the width.

For comparing the theoretical values provided by the formula with simulations, we fix some parameters and we make vary the length and width of the environment. We consider a fixed density of stations within the environment ($\delta = 0.0008$: 80 stations for a 1000×1000 square). The signal propagation radius ($r = 80$) is the same for all stations. Each reported point on the curve corresponds to 50 runs. As it can be seen on Figure 4, the curves perfectly match, for both a square (Figure 4a) or a rectangular environment (Figure 4b), meaning that our approach is relevant. With a constant density δ, the size of the environment affects directly the mean degree $i.e.$ the larger the environment is, the closer to the theoretical value for border-free environments, the mean degree is. The impact of the environment is also related to the loss of coverage of nodes which are close to the borders (area 2).

From now, and for the rest of this paper, we will consider only square environments of size $l \times l$ for simplifying the formula.

IV. Environments with One Obstacle

A. New Zones

We consider square environments in which one obstacle is positioned. For sake of clarity, and without loss of generality, the considered obstacle is itself a square of surface $S_{obs} = a^2$ inside which nodes cannot be positioned. From the station's coverage point of view, the addition of an obstacle in the environment leads to three new coverage regions $zone$ 5, $zone$ 6 and $zone$ 7. The last one is not strictly speaking a new zone since the coverage in this zone is computed in the same way as it is done for $zone$ 2. However, we introduce this new "sub"-area for simplifying the computations (see Figure 5).

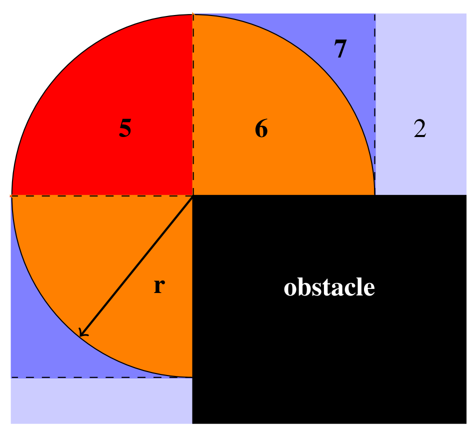


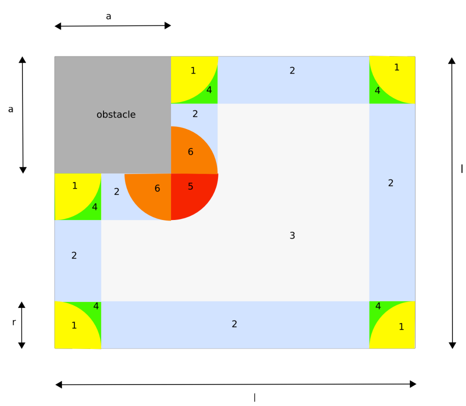
Fig. 5: New zones: zone 5, 6 and 7.

For the nodes close to the obstacle we have to calculate the coverage of a node positioned in one of these zones. For $zone$ 6 the coverage is constrained by the line-of-sight nature of the signal propagation. We precise that the nodes cannot move through the obstacle and the coverage of the nodes cannot diffuse through the obstacle. Nothing change for the first four zones.

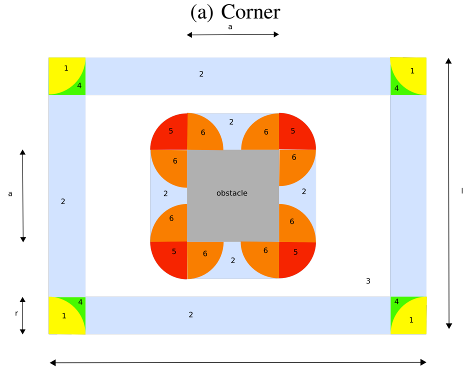
Zone 7 is part of zone 2, however, for the clarity of the formula and for an easiest expression of the mean degree we have introduce this new zone. However, the way of computing the coverage of a node in this seventh zone is the same as for *zone* 2. As it was previously done for the first four zones, we determine the mean coverages for these new zones:

$$\begin{aligned}\overline{C}_5 &= r^2(2\pi^2-1)/(2\pi)\\ \overline{C}_6 &= r^2(3\pi^2+1)/(4\pi)\\ \overline{C}_7 &= r^2(48\pi-32-9\pi^2)/(12(4-\pi))\end{aligned}$$

According to the position of the obstacle in the environment (top, corner or in the middle), each type of zone is not equaly represented. Indeed, as we can see on Figure 6, when the obstacle is centered we have four occurences of zone 5 while it appears only once when the obstacle is positionned in the corner. Thus, the mean degree depends on the number of occurrences of each zone in the environment, thus on the position of the obstacle.



(a) Corner



(b) Center

Fig. 6: Different positions for the obstacles with respect to the borders

Let us now consider formula 3 and applied it for this type of environment with one obstacle:

$$D_1 = \frac{\delta}{S}\left[\overline{C_1}S_1+\overline{C_2}S_2+\overline{C_3}S_3+\overline{C_4}S_4+\overline{C_5}S_5+\overline{C_6}S_6+\overline{C_7}S_7\right]$$

The elementary surfaces s_1, s_4, s_5, s_6 and s_7 of zones respectively 1, 4, 5, 6 and 7 do not change regardless of the position of the obstacles in the environment. The elementary surfaces s_2 and s_3 of the zones 2 and 3 depend of the size and location of the obstacles. s_2 depends on the total length of the perimeters for both the obstacles and the borders of the environment, while s_3 is equal to the surface of the environment from which are deduced the sum of the surfaces of the obstacles and the sum of all zones i with $i \in \{1,4,5,6,7\}$. The elementary surfaces s_1 and s_4 are the same as in the environment without obstacle. The elementary surfaces of the zones 5, 6 and 7 are given below:

$$\begin{aligned}s_5 &= \pi r^2/4\\ s_6 &= \pi r^2/4\\ s_7 &= r^2(4-\pi)/4\end{aligned}$$

Let $S_o = a^2$ be the surface of the obstacle. Let D_j^k be the mean degree of the environment such as k is the position of the obstacle and j the number of obstacles contained in the environment. When the obstacle is positioned in the corner, we have for every zone i ($1 \leq i \leq 7$), the following total surfaces S_i:

$$\begin{aligned}S_1 &= 5s_1 &&= 5\pi r^2/4\\ S_2 &= s_2 &&= 4rl-12r^2\\ S_3 &= s_3 &&= l^2-4rl-a^2-\pi r^2/4+5r^2\\ S_4 &= 5s_4 &&= 5r^2(4-\pi)/4\\ S_5 &= s_5 &&= \pi r^2/4\\ S_6 &= 2s_6 &&= \pi r^2/2\\ S_7 &= 2s_7 &&= r^2(4-\pi)/2\end{aligned}$$

From that we can compute the theoretical value of the mean degree when the obstacle is in the corner of the environment:

$$D_1^{corner} = \frac{\delta}{l^2-a^2}\left[\pi r^2(l^2-a^2)-(8r^3l)/3+(5r^4)/8\right]$$

Some experiments have been performed for comparing the theoretical curve with the results obtained from the simulation. For the experiments we have chosen a square obstacle of size 200×200, a transmission radius equal to $r = 80$ and a density equal to $\delta = 0.0008$ (Fig. 7)

When the obstacle is positioned at the top or away from the borders (e.g. in the center of the environment), we can apply the same computation process and we obtain:

$$\begin{aligned}D_1^{top} &= \frac{\delta}{l^2-a^2}\left[\pi r^2(l^2-a^2)-4r^3(2l+a)/3+3r^4/4\right]\\ D_1^{center} &= \frac{\delta}{l^2-a^2}\left[\pi r^2(l^2-a^2)-8r^3(l+a)/3+r^4/2\right]\end{aligned}$$

From the formula we can deduce that the impact of one obstacle on the mean degree of the graph underlying the ad hoc network is not exactly the same and depends on its position. In particular we have: $D_1^{corner} > D_1^{top} > D_1^{center}$, thus when the obstacle is in the middle of the environment its impact on the mean degree of the graph is maximized.

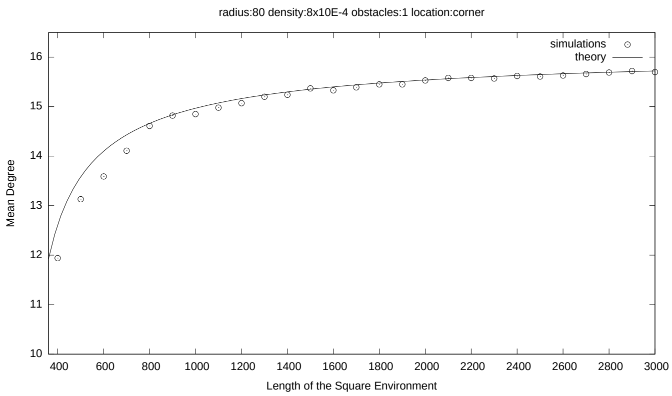
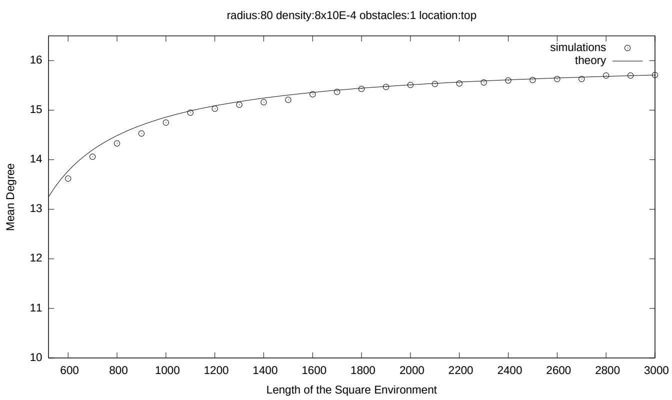
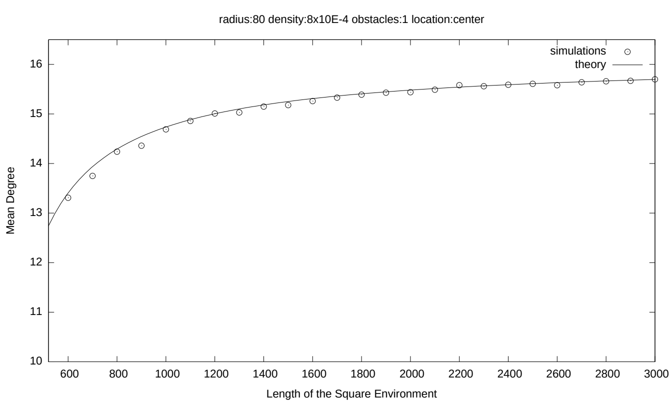


Fig. 7: Evolution of the mean degree for an environment containing one obstacle in the corner when the size of the environment is growing.



(a) Top



(b) Center

Fig. 8: Evolution of mean degree when the obstacle is in the top and in the center of the environment

In complement of the theoretical study, simulations were performed as for the corner case (square obstacle of size 200×200, $r = 80$ and $\delta = 0.0008$). The results show that the theoretic values are very close to the experimental ones. From the impact point of view, we can draw the same conclusions from the experiments: the impact of the obstacle on the mean degree of the graph is maximized when the obstacle is away from the borders.

V. Generalization

In the two previous sections we have shown that environments with obstacles may be decomposed, from the coverage point of view, in 7 zones. This decomposition depends neither on the number of obstacles, nor on their size, nor on their shape (with the restriction that obstacles have to result from overlapping rectangles), but the mean degree of the underlying graph depends on the number of occurences of each zone. In this section, we show how to compute this mean degree for different situations: when obstacles are organized in a regular grid which corresponds to a urban model and when the obstacles are not simple rectangles or square but resulting from overlapping rectangles.

A. Grid-based Organization of Obstacles

We consider a set of square obstacles organized as a grid, such that no obstacle is close to the borders. Obstacles are embedded in an environment of size $l \times l$. The obstacles are of equal size $a \times a$. As previously mentionned, the distance between obstacles and between obstacles and the borders of the environment has to be greater than Δ that represents twice the value of the transmission range. This situation is represented by Figure 9.

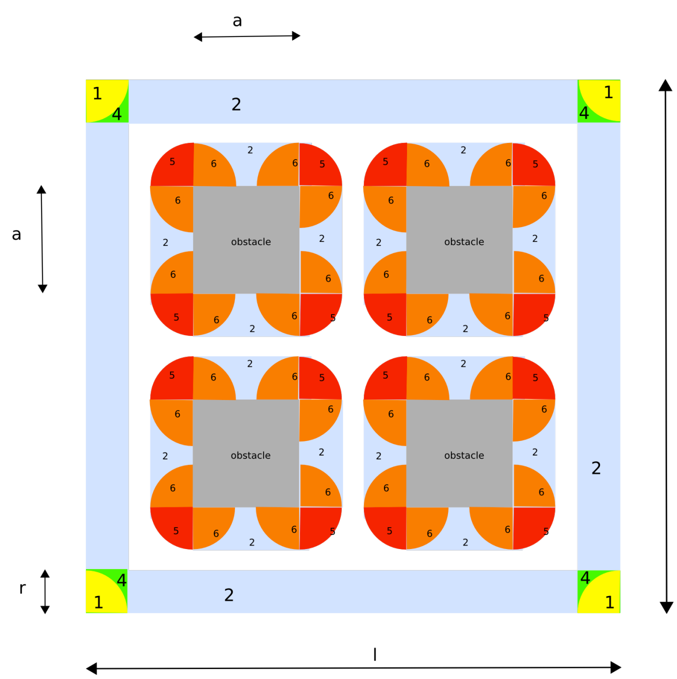


Fig. 9: Environments containing four obstacles organized as a grid.

As it has been done several times in the previous sections, we can express the theoretical mean degree by counting the number of occurrences of each zone since the mean value of the coverage is invariant. From this process we obtain the following formula:

$$D_4^{grid} = \frac{\delta}{l^2 - 4a^2}\left[\pi r^2(l^2 - 4a^2) - \frac{4r^3}{3}(2l + 8a) + \frac{r^4}{2}\right]$$

We compare this theoretical value with some experiments fixing the size of the obstacles to 200×200, the density to $\delta = 0.0008$ and the transmission radius to $r = 80$. We compute the mean degree evolution according to the size of the environment. For every computation, the number of runs is equal to 20. The results are reported on Figure 10. As it was the case without obstacle and with only one obstacle we can observe that both curves (the theoretical one and the experimental one) are very close.

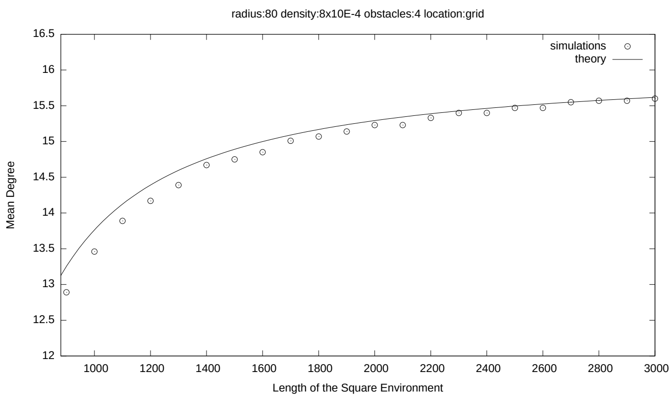


Fig. 10: Evolution of the mean degree: square environment with four obstacles.

B. Irregular Organization and Shape of Obstacles

The particularity of the formulation presented along this paper is that it allows the computation of the theoretical value of mean degree for any environment containing obstacles made of overlapping rectangles. For that it is enough to count the number of occurrences of each zone and to measure the perimeter of both the obstacles and of the borders of the environment. Thus, it is also possible, from the mean degree point of view, to build environments with obstacles that are mean-degree-equivalent.

We place four obstacles identified from A (1) to D (4) in the environment (Fig. 11). Every obstacle own different characteristics and has its own position in the environment. The obstacle A ($L_1 \times l_1$) is in the corner, obstacle B ($L_2 \times l_2$) can be considered as placed in the center of the environment, while obstacle C ($L_3 \times l_3$) is stuck to the border of the environment. Finally obstacle D is composed of two overlapping rectangles D' ($L'_4 \times l'_4$) and D'' ($L''_4 \times l''_4$). Let us study this particular case:

- number of occurrences of zone 1: 8
- the length of the zone 2: $2(L+l) + 2L_3 + 2(l'_4 + l'_4) + 2L''_4 - l''_4 - 26r$
- number of occurrences of zone 4: 8
- number of occurrences of zone 5: 12
- number of occurrences of zone 6: 24
- number of occurrences of zone 7: 24

The formula for computing the theoretical value of the mean degree remains unchanged:

$$D_4^{\text{random}} = \frac{\delta}{S}\left[8\overline{C_1}s_1 + \overline{C_2}s_2 + \overline{C_3}s_3 + 8\overline{C_4}s_4 + 12\overline{C_5}s_5 + 24\overline{C_6}s_6 + 24\overline{C_7}s_7\right]$$

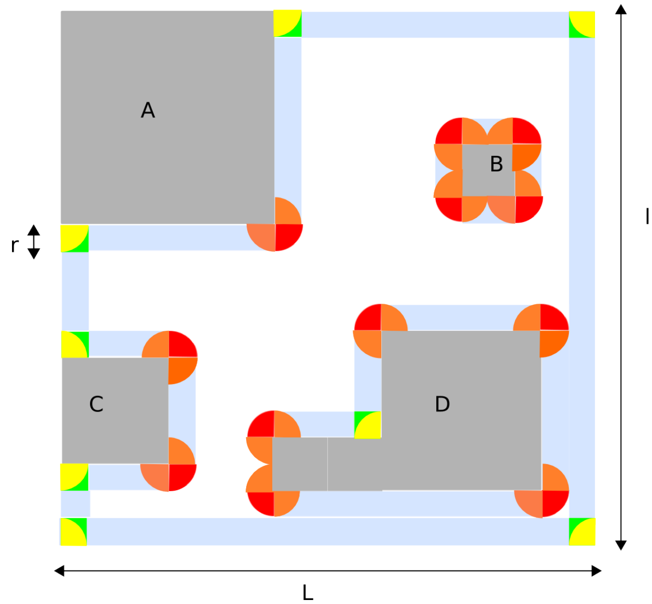


Fig. 11: A situation with four obstacles variously positioned in the environment.

We can notice that different environments with obstacles may have the same formula for the theoretical mean degree. If we consider a new square environment with two obstacles in contact with the borders and two square obstacles in the middle, we have the same number of occurrences for zones 1 and 4 and 5 and 6, the size of the obstacles have to be chosen such that the total perimeter (obstacles+borders) and the total surface of the obstacles are both equal in the two scenarios.

VI. Conclusion and Perspectives

The environment plays an key role for the computing of the mean degree. Most of simulators integrating obstacles do not take into account the impact of borders on the mean degree of stations. The idea mostly used in mobile ad hoc networks is that radio waves can go through walls without be impacted by them. Environment borders or obstacles impacts only the moving of stations. By considering a particular type of environment and obstacle, we have showed that the impact of the environment on the mean degree can be approximated analytically and depends on five main parameters: The surface of the environment, the surface of obstacles, the perimeter of environment, the perimeter of obstacles and the position of the obstacles. From this calculation, it is possible to introduce attenuation functions on the coverage areas around obstacles and borders. An other obtained result is that more there are isolated nodes in the environment and less the theoretic mean degree is closed to the simulated mean degree.

References

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