Maintaining Shortest Paths in Dynamic Graphs

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For a long time, the problem of computing a shortest path in a dynamic graph has been received a lot of attention, mainly because of its crucial importance for application domains like logistics and transportation. However, different problems may be distinguished according to the assumptions. There exist at least three classes of problems depending on the way dynamics is considered. The first set of problems relies on the notion of Time-Dependent Networks. The considered model is a directed graph which temporal characteristics are fully known before processing the graph. Several interesting results have already been proved. For instance, Kaufmann and Smith have shown that finding the minimum time walk in FIFO networks is polynomial [2]. But, if the FIFO constraint is not satisfied, the problem becomes NP-Hard [3]. However, for this kind of problems, whatever the difficulty is (solvable in polynomial time or NP-hard), classical static approaches may be used since it is possible to build, from the original graph and the set of future time-stamped modifications, a time-expanded graph, like the Space-Time Networks and Chrono-SPT [4].

The second class of problems relies on controllable dynamics of graphs. As a consequence, between two sets of changes in the graph (edge insertion/deletion, weight changes), the algorithm that operates on the modified graph may take as long as required for its completion. The goal is then to conceive algorithms that minimize the time needed to recover the shortest paths. Re-optimization technics are well-suited for that task. The idea is to use the previous solution in order to find the next one instead of restarting the optimization from scratch. When the changes are small, one can typically reconstruct the solution with less efforts. Many different approaches from this class have been proposed in the last couple of years, handling different types of changes in the graph more or less efficiently. These approaches are based on some combinatorial and algebraic properties of the graphs, as well as on efficient data structures. The reader is referred to [1] for a detailed survey on them. In this paper we propose a new re-optimization algorithm based on the network simplex method. We consider the more general minimum cost flow (MCF) problem. which consists in finding a minimum cost flow on the arcs satisfying demand and supply constraints on the nodes. The network simplex method is an adaption of the primal simplex algorithm, specifically for the MCF problem. The basis is represented as a rooted spanning tree of the underlying graph, in which variables are represented by arcs. The method iterates towards an optimal solution by exchanging basic and non-basic arcs. At each iteration, an entering arc is selected by some pricing strategy. The entering arc forms a cycle in the spanning tree and one of the arcs of this cycle is selected to leave the basis. Each modification in the graph may break either the primal or the dual feasibility of the current basis. In this case we apply either dual or primal simplex network method starting from this basis. The use of appropriate data structures ensures fast pricing and pivoting. Theoretically, the number of simplex iterations needed to restore the basis may even be exponential, but in practice this number turns to be very small. Our tests show that the number of iterations is practically constant and limited to 2-3 even for graphs with as many as 5000 nodes and 20000 edges.

Finally the last class gathered problems for which the dynamics is not controlled. Graph modifications may occur without prior notification. Between two sets of graph modification the algorithm that searches for shortest paths operates during a limited time that do not depend on the number of vertices and/or edges. In such a framework, the goal consists in trying to maintain a solution as close as possible from the optimal one. This kind of situation occurs within decentralized and dynamic environments like mobile ad hoc networks. In such networks, not only dynamics is a constraint but also the distributed characteristic of the system. Thus, dynamics is not controlled and the graph itself may not be considered globally. The need of algorithms to compute shortest paths in these networks is close to routing purpose. The deferent constraints of MANETs impose algorithms to be robust and anytime. *Robust* because the dynamic of the network is not controlled, it is impossible to forecast the coming of events. The algorithm cannot stop the evolution of the graph *Anytime* because at any moment, as soon as the network is connected, a solution should be available. The query for a shortest path to the algorithm may appear at any moment during the computation of the algorithm. In this paper we propose a method based on the collective intelligence paradigm and more precisely on ant colonies. Ants are able to find a shortest path from the nest to a food source without any centralized control. The copy of such a mechanism is well suited in the field of MANETs and it turns out to be robust to topological changes.

References

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