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TOPOLOGY IDENTIFICATION OF COMPLEX DYNAMICAL NETWORK WITH HINDMARSH-ROSE NEURONS

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Abstract. Complex networks, such as transportation and phone networks, Internet, wireless networks and the World Wide Web, play an important role in our life nowadays. Significant progress has been made in studying complex networks since the discovery of their small-world [1] and scale-free[2] characteristics.

Synchronization, as an interesting phenomenon of a population of dynamically interacting units, has received wide attention from different fields. Assuming the topology of complex network known, many researchers discuss the synchronization problem of complex network and obtain a lot of important results. For example, Pecora and Carroll studied the master stability functions for synchronized coupled systems[3]. Using the theory of inhomogeneous Markov chains, Wu proposed a synchronization criterion for nonautonomous discrete-time linear system in random directed network [4]. Belykh *et al.* proposed the synchronization of coupled chaotic systems via connection graph stability method [5, 6].

The topology identification, as an inverse problem, is a significant issue in the study of complex networks. For example, if a major malfunction occurs in a communication network, power network or the Internet, it is very important to quickly detect the location of the faulty line. Topology identification of complex dynamical networks has received more and more attention from the systems science community. Generally speaking, two methods are used to study the identification problem [7]-[14]. One is based on the classic adaptive control. Considering the complex dynamical network with unknown topology as a drive system, some researchers built a response system and adaptive controls to estimate the unknown topology. The other mainly used method consists on solving large scale linear equation. Driving the complex dynamical network to an equilibrium state by a control law, the identification problem can be transformed to solve the large linear equation. It is worth remarking that the earlier results neglected a crucial condition: persistent excitation (PE) condition [15]. The condition is so important that the topology cannot be identified successfully without it.

In 1952, two neurophysiologists proposed a mathematical model that describes neuron activity [16]. This model have been modified into different other models. In this study, we focus on one of them, the Hindmarsh-Rose model (HR), which can exhibit most of biological neuron behavior, such as spiking, bursting [17, 18]. Meanwhile, it is also important to investigate the group action when they are coupled each other.

Let us consider a network composed by n HR neurons. These neurons are coupled by their first variable x_i, which described by

$$\begin{cases} \dot{x}_i = ax_i^2 - x_i^3 + y_i - z_i - \sum_{j=1}^{n} c_{ij} h(x_i, x_j), \\ \dot{y}_i = (a+\alpha)x_i^2 - y_i, \\ \dot{z}_i = \epsilon(bx_i + c - z_i), \end{cases} \tag{1}$$

for $i = 1, ..., n$, where h is the coupling function and $C = c_{ij}$ is the network adjacency matrix. We assume that the elements of the matrix C are unknown. When the neurons are chemically coupled, the coupling function h is given by

$$h(x_i, x_j) = k\frac{x_i - V}{1 + exp(-\lambda(x_j - \Theta))}, \tag{2}$$

where k is the coupling strength, Θ is the threshold reached by every action potential for a neuron and V is the reversal potential.

Recently, Chenhui Jia *et al* designed a bridging network and adaptive law to estimate the topology of systems (1), and give many significant results. However, these results seem questionable, since they neglected the PE condition. In this paper, we estimate these unknown parameters in an asymptotic manner and discuss the important of PE condition on the process of identification. Pinning control is used to drive the response system synchronize with the drive system because it is difficult to control all the nodes of the large network. Additionally, we consider the topology identification of complex network with noise as many real systems will be subject to perturbation.

Keywords. Complex network, Topology identification, Synchronization, Persistent excitation.

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